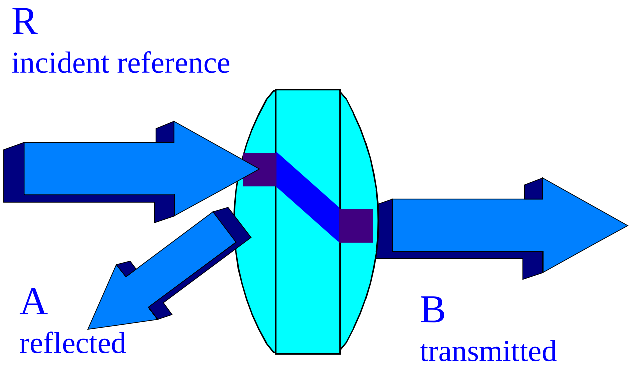
3.3.3.1. BASICS OF S-PARAMETERS, part 1

C O N T E N T S of part 1:

THE DEFINITION OF THE S-PARAMETERS



Scatter Parameters, also called S-parameters, belong to the group to twoport parameters used in twoport theory. Like the Y or Z parameter, they describe the performance of a twoport completely. Different to Y and Z, however, they relate to the traveling waves that are scattered or reflected when a network is inserted into a transmission line of a certain characteristic impedance Z_L. Therefore, S-parameters can be compared to reflection and throughpass of a pair of spectacles.

S-parameters are important in microwave design because they are easier to measure and to work with at high frequencies than other kinds of twoport parameters. They are conceptually simple, analytically convenient and capable of providing detailed insight into a measurement and modeling problem. However, it must kept in mind that -like all other twoport parameters, S-parameters are linear by default. I.e. they represent the *linear* behavior of the twoport.

Staying with the pair of spectacles examples for a moment, i.e. power-wise, the S-parameters are defined as:

$$\begin{pmatrix} |\underline{b}_1|^2 \\ |\underline{b}_2|^2 \end{pmatrix} = \begin{pmatrix} |\underline{S}_{11}|^2 & |\underline{S}_{12}|^2 \\ |\underline{S}_{21}|^2 & |\underline{S}_{22}|^2 \end{pmatrix} * \begin{pmatrix} |\underline{a}_1|^2 \\ |\underline{a}_2|^2 \end{pmatrix}$$

with

$|\underline{a}_i|^2$	power wave traveling towards the twoport gate
$|\underline{b}_i|^2$	power wave reflected back from the twoport gate

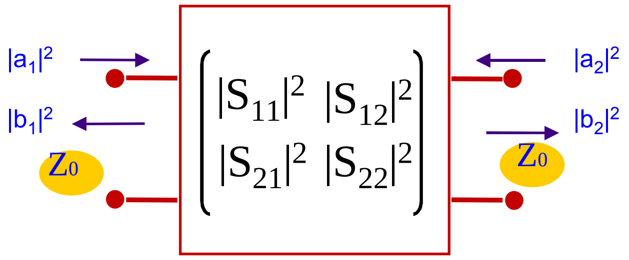
and

$|\underline{S}_{11}|^2$	power reflected from port1
$|\underline{S}_{12}|^2$	power transmitted from port1 to port2
$|\underline{S}_{21}|^2$	power transmitted from port2 to port1
$|\underline{S}_{22}|^2$	power reflected from port2

Note: $\underline{a}_i$, $\underline{b}_i$ and $\underline{S}_{ij}$ are effective values and not peak values of the corresponding sine functions.

This means that S-parameters do relate traveling waves (power) to a twoport's reflection and transmission behavior. Since the twoport is imbedded in a characteristic impedance of Z_0, these 'waves' can be interpreted in terms of normalized voltage or current amplitudes. This is explained below.

S-Parameters and Characteristic Impedance Z0



Starting with power | normalized to Z0 | gives normalized amplitudes for voltage and current

$$P = v^*i \quad = \quad \frac{v^*v}{Z_0} \quad \longrightarrow \quad \sqrt{P} = \frac{V}{\sqrt{Z_0}} = i^*\sqrt{Z_0}$$

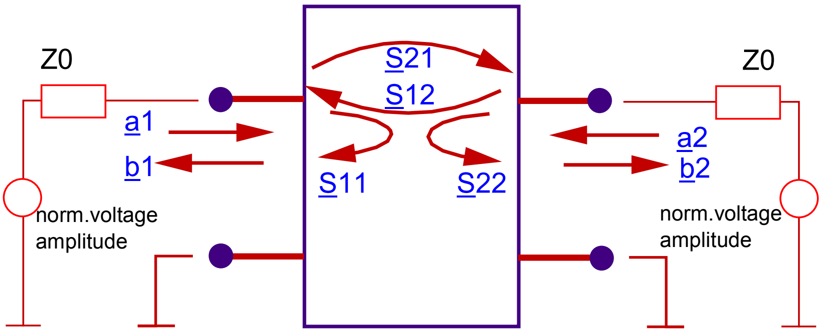
In other words, we can convert the power towards the twoport into a normalized voltage amplitude of

$$a_i = \frac{V_{towards\_twoport}}{\sqrt{Z_0}} \tag{1}$$

and the power away from the twoport can be interpreted in terms of voltages like

$$b_i = \frac{V_{away\_from\_twoport}}{\sqrt{Z_0}} \tag{2}$$

The following signal flow graph gives the situation for the S-parameter interpretation in voltages:



Looking at the S-parameter coefficients individually, we have:

$$\underline{S}_{11} = \frac{\underline{b}_1}{\underline{a}_1} = \frac{V_{\text{reflected at port1}}}{V_{\text{towards port1}}}\Bigg/_{\underline{a}_2 = 0} \qquad \underline{S}_{12} = \frac{\underline{b}_1}{\underline{a}_2} = \frac{V_{\text{out of port1}}}{V_{\text{towards port2}}}\Bigg/_{\underline{a}_1 = 0}$$

$$\underline{S}_{21} = \frac{\underline{b}_2}{\underline{a}_1} = \frac{V_{\text{out of port2}}}{V_{\text{towards port1}}}\Bigg/_{\underline{a}_2 = 0} \qquad \underline{S}_{22} = \frac{\underline{b}_2}{\underline{a}_2} = \frac{V_{\text{reflected at port2}}}{V_{\text{towards port2}}}\Bigg/_{\underline{a}_1 = 0} \qquad (3)$$

S11 and S21 are determined by measuring the magnitude and phase of the incident, reflected and transmitted signals when the output is terminated in a perfect Zo load. This condition guarantees that a2 is zero. S11 is equivalent to the input complex reflection coefficient or impedance of the DUT, and S21 is the forward complex transmission coefficient.

Likewise, by placing the source at port 2 and terminating port 1 in a perfect load (making a1 zero), S22 and S12 measurements can be made. S22 is equivalent to the output complex reflection coefficient or output impedance of the DUT, and S12 is the reverse complex transmission coefficient.
The accuracy of S-parameter measurements depends greatly on how good a termination we apply to the port not being stimulated. Anything other than a perfect load will result in a1 or a2 not being zero (which violates the definition for S-parameters). When the DUT is connected to the test ports of a network analyzer and we don't account for imperfect test port match, we have not done a very good job satisfying the condition of a perfect termination. For this reason, two-port error correction, which corrects for source and load match, is very important for accurate S-parameter measurements.

Let's now discuss some characteristic S-parameter values.

S11 and S22

value	interpretation
-1	all voltage amplitudes towards the twoport are inverted and reflected (0 Ω)
0	impedance matching, no reflections at all (50 Ω)
+1	voltage amplitudes are reflected (infinite Ω)

The magnitude of S11 and S22 is always less than 1. Otherwise, it would represent a negative ohmic value. On the other hand, the magnitude of S21 (transfer characteristics) respectively S12 (reverse) can exceed the value of 1 in the case of active amplification. Also, S21 and S12 can be positive an negative. If they are negative, there is a phase shift. Example: S21 of a transistor starts usually at about S21 = -2   -10. This means signal amplification within the Z0 environment *and* phase shift.

S21 and S12

magnitude	interpretation
0	no signal transmission at all
0 ... +1	input signal is damped in the Z0 environment
+1	unity gain signal transmission in the Z0 environment
> +1	input signal is amplified in the Z0 environment

The numbering convention for S-parameters is that the first number following the S is the port at which energy emerges, and the second number is the port at which energy enters. So S21 is a measure of power emerging from Port 2 as a result of applying an RF stimulus to Port 1.

In order to better understand the Sxx parameters, let's consider an ideal transmission line, connected to a port of the VNA, and terminated with either a LOAD, an OPEN or a SHORT, or an ideal RESISTOR. (This chapter is from HP App.Note 1287-1).

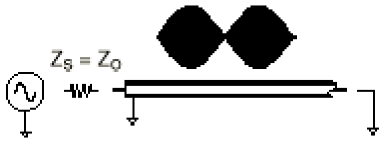


Fig.1: a transmission line connected to a VNA

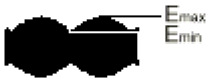
At very low frequencies, with wavelengths much larger than the line, the transmission line can be thought of a simple wire. This is adequate for conducting DC or very low frequency power. The resistance of the wire is relatively low and has little effect on low-frequency signals. The voltage and current are the same no matter where a measurement is made on the wire. At higher frequencies, wavelengths are comparable to or smaller than the length of the transmission line (or a conductor in a high-frequency circuit), and power transmission can be thought of in terms of traveling waves. When the transmission line is terminated in its characteristic impedance, maximum power is transferred to the load. When the termination is not equal to the characteristic impedance, that part of the signal that is not absorbed by the load is reflected back to the source.

If a transmission line is terminated in its characteristic impedance Z0 (LOAD condition), no reflected signal occurs since all of the transmitted power is absorbed by the load. Looking at the envelope of the RF signal versus distance along the transmission line shows no standing waves because without reflections, energy flows in only one direction.

When the transmission line is terminated in a SHORT circuit (which can sustain no voltage and therefore dissipates zero power), a reflected wave is launched back along the line toward the source. The reflected voltage wave must be equal in magnitude to the incident voltage wave and be 180 degrees out of phase with it at the plane of the load. The reflected and incident waves are equal in magnitude but traveling in the opposite directions.

If the transmission line is terminated in an OPEN-circuit condition (which can sustain no current), the reflected current wave will be 180 degrees out of phase with the incident current wave, while the reflected voltage wave will be in phase with the incident voltage wave at the plane of the load. This guarantees that the current at the open will be zero. The reflected and incident current waves are equal in magnitude, but traveling in the opposite directions. For both the short and open cases, a standing wave pattern is set up on the transmission line. The voltage valleys will be zero and the voltage peaks will be twice the incident voltage level.

If the transmission line is terminated with say an ideal 25 Ω RESISTOR, resulting in a condition between full absorption and full reflection, part of the incident power is absorbed and part is reflected. The amplitude of the reflected voltage wave will be one-third that of the incident wave, and the two waves will be 180 degrees out of phase at the plane of the load. The valleys of the standing-wave pattern will no longer be zero, and the peaks will be less than those of the short and open cases. The ratio of the peaks to valleys will be 2:1.



Voltage Standing Wave Ratio

$$\mathbf{VSWR} = \frac{E_{max}}{E_{min}} = \frac{1+\rho}{1-\rho}$$

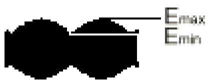
Fig.2: the voltage standing wave ratio

The traditional way of determining RF impedance was to measure VSWR using an RF probe/detector, a length of slotted transmission line, and a VSWR meter. As the probe was moved along the transmission line, the relative position and values of the peaks and valleys were noted on the meter. From these measurements, impedance could be derived. The procedure was repeated at different frequencies. Modern network analyzers measure the incident and reflected waves directly during a frequency sweep, and impedance results can be displayed in any number of formats (including VSWR).

The most general term for ratioed reflection is the complex reflection coefficient, Γ (gamma). The magnitude portion of Γ is called ρ (rho). The reflection coefficient is the ratio of the reflected signal voltage level to the incident signal voltage level. For example, a transmission line terminated in its characteristic impedance Z0, will have all energy transferred to the load so Vrefl = 0 and ρ = 0. When the impedance of the load, ZL is not equal to the characteristic impedance, energy is reflected and ρ is greater than zero. When the load impedance is equal to a short or open circuit, all energy is reflected and ρ = 1. As a result, the range of possible values for ρ is 0 to 1.

Reflection Coefficient $\Gamma = \frac{V_{reflected}}{V_{incident}} = \rho\angle\Phi = \frac{Z_L - Z_O}{Z_L + Z_O}$

Return loss $= -20\log(\rho),\ \rho = |\Gamma|$



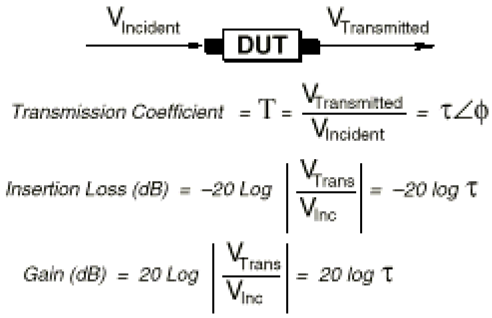
Voltage Standing Wave Ratio

VSWR $= \frac{E_{max}}{E_{min}} = \frac{1+\rho}{1-\rho}$

Fig.3: Reflection Parameters

Return loss is a way to express the reflection coefficient in logarithmic terms (dB). Return loss is the number of decibels that the reflected signal is below the incident signal. Return loss is always expressed as a positive number and varies between infinity for a load at the characteristic impedance and 0 dB for an open or short circuit. Another common term used to express reflection is voltage standing wave ratio (VSWR), which is defined as the maximum value of the RF envelope over the minimum value of the RF envelope. It is related to ρ as (1 + ρ)/(1 – ρ).
VSWR ranges from 1 (no reflection) to infinity (full reflection).

After these thoughts on the Sxx parameters, lets finish by considering the properties of the Sxy parameters. The transmission coefficient is defined as the transmitted voltage divided by the incident voltage. If the absolute value of the transmitted voltage is greater than the absolute value of the incident voltage, a DUT or system is said to have gain. If the absolute value of the transmitted voltage is less than the absolute value of the incident voltage, the DUT or system is said to have attenuation or insertion loss. The phase portion of the transmission coefficient is called insertion phase.



Fig,4: Transmission parameters

Direct examination of insertion phase usually does not provide useful information. This is because the insertion phase has a large (negative) slope with respect to frequency due to the electrical length of the DUT. The slope is proportional to the length of the DUT. Since it is only deviation from linear phase that causes distortion in communications systems, it is desirable to remove the linear portion of the phase response to analyze the remaining nonlinear portion. This can be done by using the electrical delay feature of a network analyzer to mathematically cancel the average electrical length of the DUT. The result is a high-resolution display of phase distortion or deviation from linear phase.

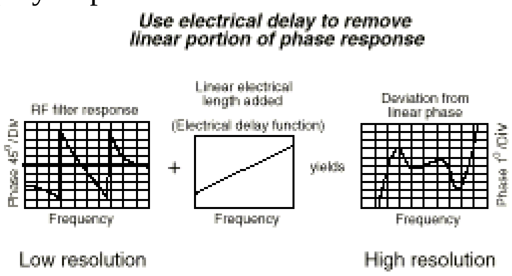


Fig.5: Applying electrical delay

Note: adding electrical delay should not be used for modeling!

Another useful measure of phase distortion is group delay. This parameter is a measure of the transit time of a signal through a DUT versus frequency. Group delay can be calculated by differentiating the DUT's phase response versus frequency. It reduces the linear portion of the phase response to a constant value, and transforms the deviations from linear phase into deviations from constant group delay, (which causes phase distortion in communications systems). The average delay represents the average signal transit time through a DUT.

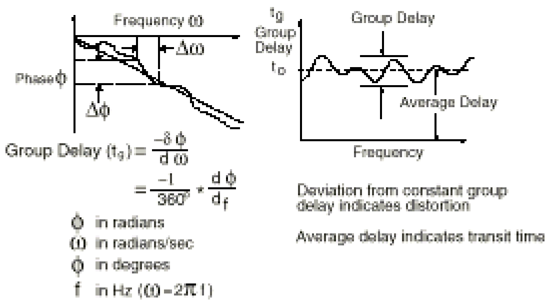


Fig.6: Understanding group delay

UNDERSTANDING THE SMITH CHART TRANSFORMATION

What makes S_{xx}-parameters especially interesting for modeling, is that S11 and S22 can be interpreted as complex input or output resistances of the twoport (including the termination at the opposite side of the twoport with Z0 !!).
That's why they are usually plotted in a Smith chart. This chapter is intended to explain the basics of such Smith charts.

The Smith chart is a transformation of the complex impedance plane $\underline{R}$ into the complex reflection coefficient $\underline{\Gamma}$ (rho) following:

$$\underline{\Gamma} = \frac{\underline{R} - Z0}{\underline{R} + Z0} \qquad (1)$$

with the system's reference impedance Z0 = 50 Ω.

This means that the right half of the complex impedance plane $\underline{R}$ is transformed into a circle in the $\underline{\Gamma}$-domain. The circle radius is '1' (see fig.1).

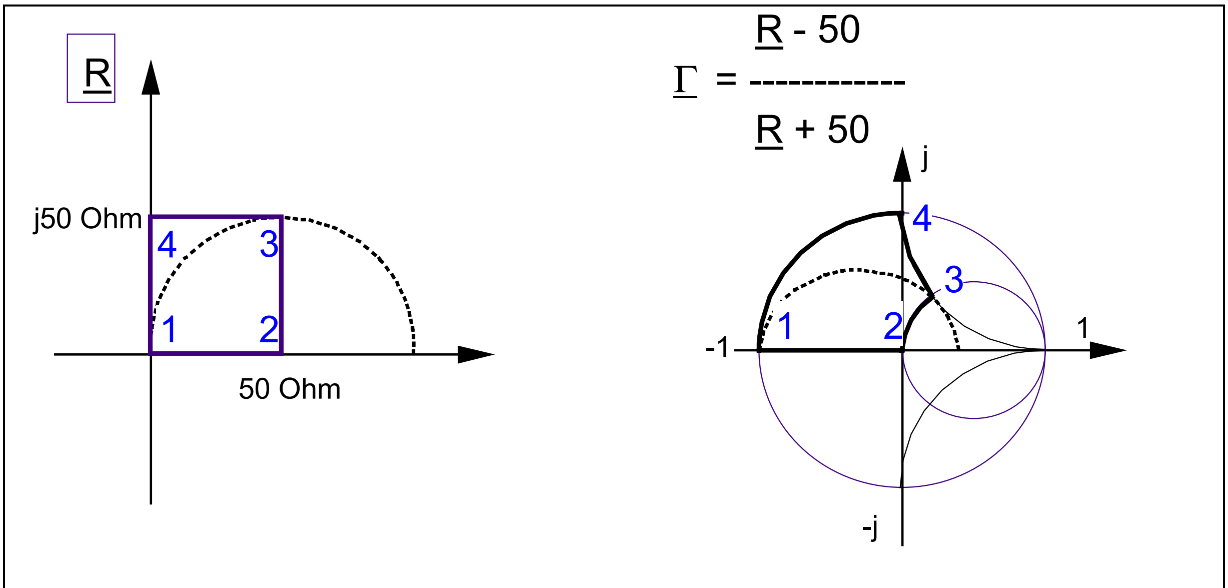


Fig.1: the relationship between Sxx and the complex impedance of a twoport.

On the other hand, using an network analyzer with a system impedance of Z0, the parameter S11 is equal to

$$S_{11} = 2 \cdot \frac{\underline{v1}}{v01} - 1 \qquad (2)$$

where $\underline{v}1$ is the complex voltage at port 1 and v01 the stimulating AC source voltage (typically normalized to '1'). See fig.2, and the following chapter on 'calculating S-parameters from complex voltages' for details.

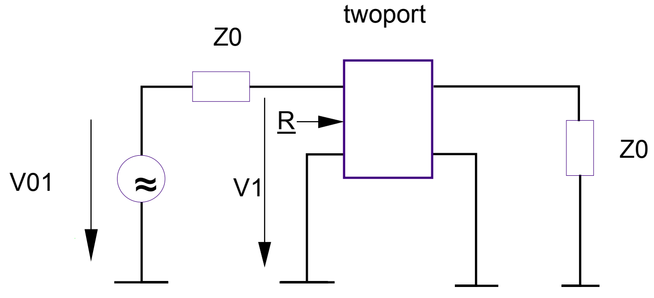


fig.2: about the definition of S11 (S22 is just analogous).

Under the assumption that $\underline{R}$ is the complex input resistance at port 1 and Z0 is the system impedance, we get using eq.(2) and the resistive divider formula:

$$\underline{S_{11}} = 2 \cdot \frac{\underline{R}}{\underline{R} + Z0} - 1 = \frac{\underline{R} - Z0}{\underline{R} + Z0}$$

And this is the reflection coefficient $\underline{\Gamma}$ from (1) !!

After all, if the reflection coefficient $\underline{\Gamma}$ resp. S_{11} or S_{22} is known, we get for the complex resistor $\underline{R}$:

$$\underline{R} = Z0 \cdot \frac{1 + \underline{\Gamma}}{1 - \underline{\Gamma}} = Z0 \cdot \frac{1 + \underline{S_{11}}}{1 - \underline{S_{11}}}$$, with usually Z0 = 50Ω

This explains how we can get the complex input/output resistance of a twoport directly from S11 or S22, if we plot these S-parameters in a Smith chart.

Let's go back to fig.1 and consolidate this context a little further:
it shows a square with the corners (0/0)Ω, (50/0)Ω, (50/j50)Ω and (0/j50)Ω in the complex impedance plane and its equivalent in the Smith chart with Z0=50Ω. Please watch the angel-preserving property of this transform (rectangles stay rectangles close to their origins). Also watch how the positive and negative imaginary axis of the $\underline{R}$ plane is transformed into the Smith chart domain ($\underline{\Gamma}$), and where (50/j50) Ω is located in the Smith chart. Also verify that the center of the Smith chart represents Z0, i.e. for Z0=50Ω, the center of the Smith chart is (50/j0) Ω.

Fig.3 explains once again the transformation of the complex ohmic plane to the Smith chart.

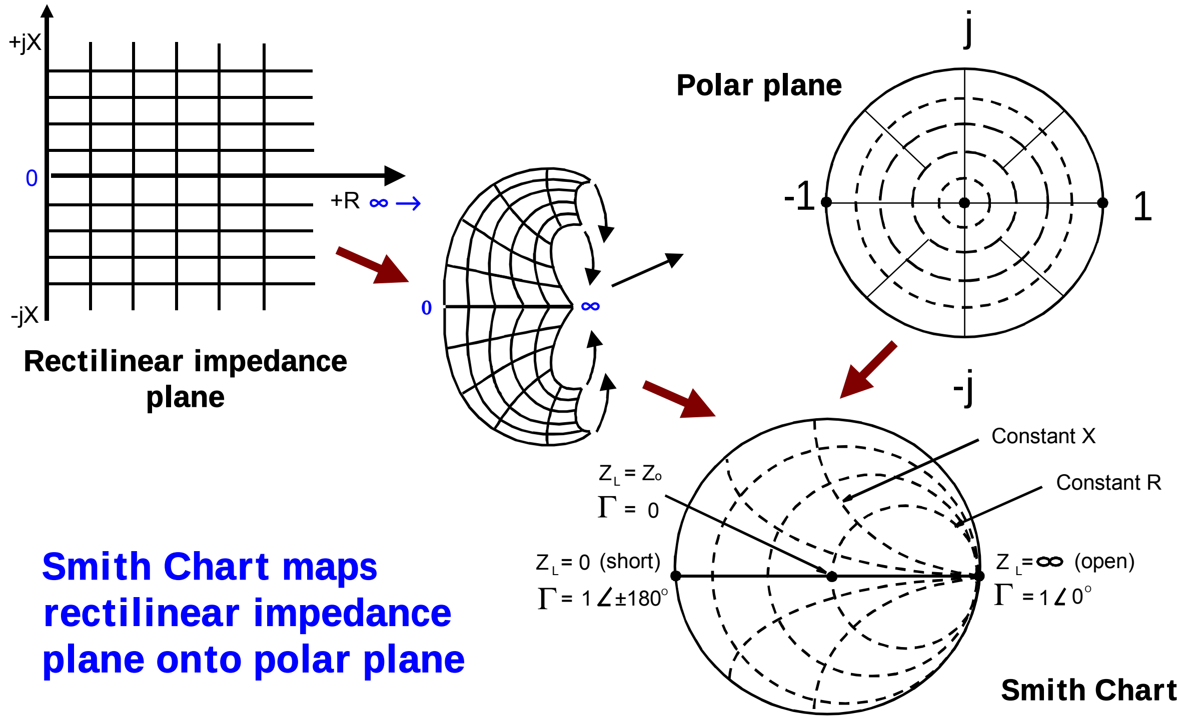
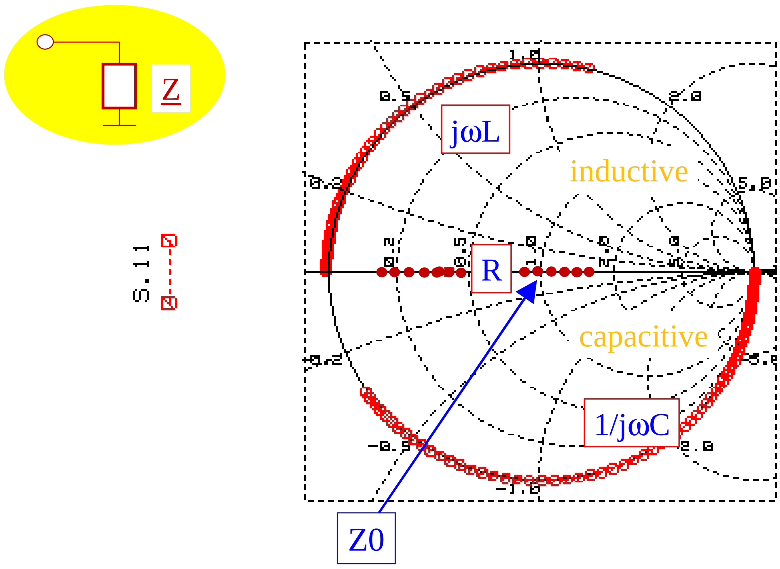


Fig. 3: how the complex resistance plane is transformed into the Smith chart

This allows us to make the following statements:

> Sxx on the real axis represent ohmic resistors
> Sxx above the real axis represent inductors
> Sxx below the real axis represent capacitors
> S_{xx} curves in the Smith chart *turn clock-wise with increasing frequency*.



As an example for interpreting Smith charts, fig.4a shows the S11 plot of a bipolar transistor.

In this case, the locus curve stars with S11 ≈ 1 = ∞*Z0 at low frequencies (RBB' + Rdiode + beta*RE). For higher frequencies, the curves then tend towards |S11| ∈ {-1 .. 0} for high frequencies (the CBE shorts Rdiode, and beta = 1. Therefore, the end point of S11 is RBB' + RE). Since RBB' is bias dependent, and decreasing with increasing iB, the end points of the curves represent this bias-dependency. For incrementing frequency, the S11 locus curve turns clockwise-

Fig.4b shows the S11 curve of a capacitor located between the NWA ports. The capacitor represents an OPEN for DC, thus S11 ~ 1 = ∞*Z0. For highest frequencies, it behaves like a SHORT, and we see the 50 Ω of the opposite port2 (!). The transition between the DC point and infinite frequency follows a circle, and the increasing frequency turns the curve again clockwise.

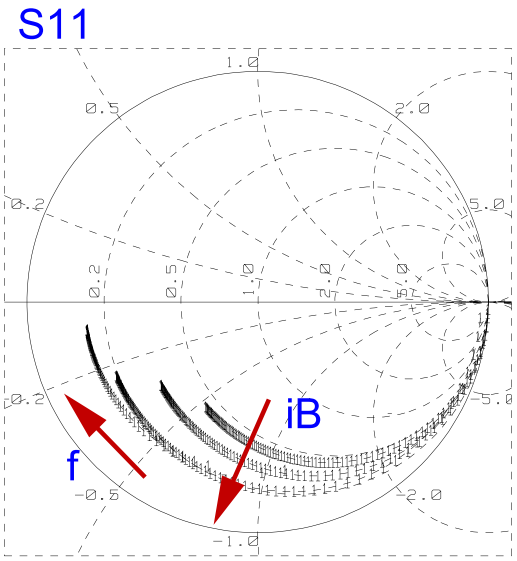


Fig.4a: S_{11} of a transistor with increasing Base current iB.

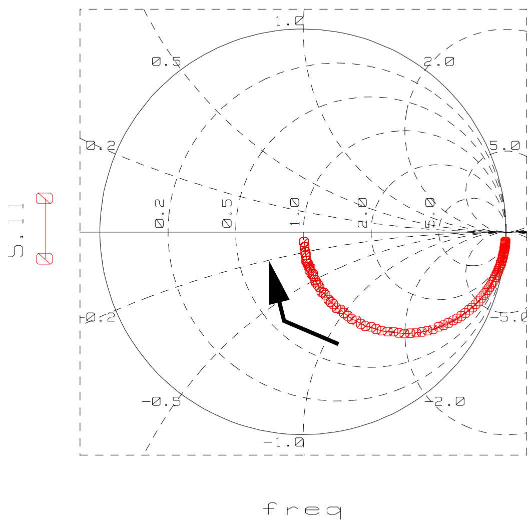


Fig4b: S_{11} of a capacitor between port 1 and port 2

UNDERSTANDING POLAR PLOTS OF S12 AND S21

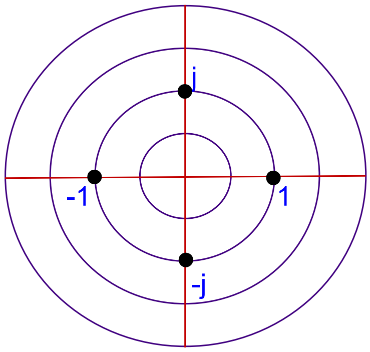


Fig.1: The polar plot for S12 and S21

The S21 parameter represents the power transmission from port 1 to port 2, if the twoport is inserted into a matching network with characteristic impedance Z0 of e.g. 50Ω. This means, if no voltage is transmitted, then S21=0 (in the center of the polar plot). If voltage is transmitted, we are on the positive X-axis. The curve will be below S21=1 for damping between the port 1 and port 2, and above S21=1 for amplification. If the phase is inverted, we are basically in the left half-plane of the polar plot (REAL[S21]<0). And, for voltage amplification, but also phase shift of e.g.-180 degrees (a transistor), below S21=-1.

Like with the Smith chart, all S21 and S12 curves *turn clock-wise with increasing frequency*.

As an example, fig.2 shows the S21 plot of a bipolar transistor, and fig.3 of a capacitor between port 1 and port 2. While the transistor starts with S21 <-1 at low frequencies (voltage amplification in a 50Ω system), its curves tend towards S21=0 for high frequencies (no voltage transmission, the transistor capacitances short all voltage transmission). For the capacitor it's just the opposite: no power transmission for lowest frequencies, but an ideal short (S21=1) for highest frequencies. For more details, see the chapter on understanding S-parameters plots.

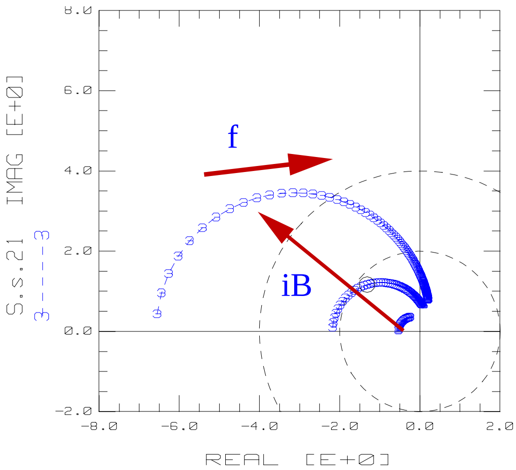


Fig.2: S21 of a transistor with varying Base current iB.

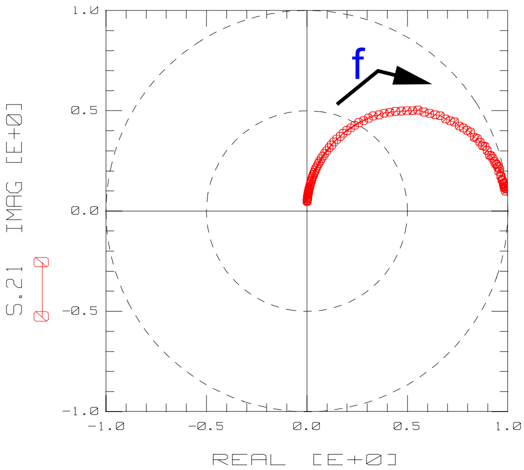


Fig3: S21 of a capacitor between port 1 and port 2

CALCULATING S-PARAMETERS FROM COMPLEX VOLTAGES

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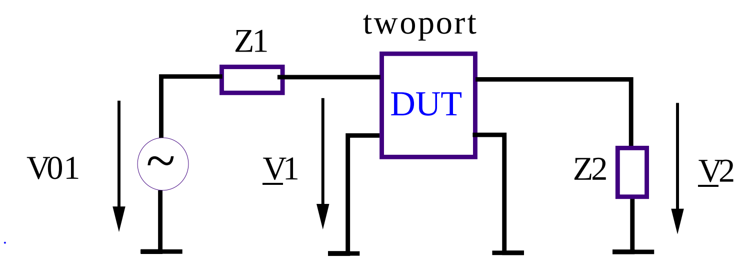
As it was mentioned before, S-parameters can be interpreted in terms of voltage at the DUT in a Z0 environment.

The following sketch gives an explanation about how to calculate them for a given twoport (DUT), imbedded in an external circuit, which itself represents the characteristic impedance.

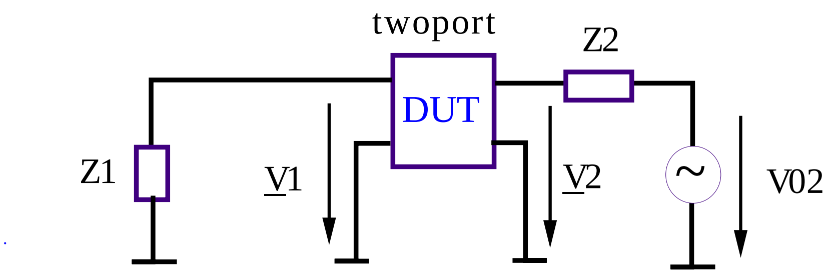
Z1 = Z2 = Z0
e.g.Z0 = 50 Ω



$$\underline{S}_{11} = 2 * \frac{\underline{V}_1}{\underline{V}_{01}} - 1 \quad (1)$$

$$\underline{S}_{21} = 2 * \frac{\underline{V}_2}{\underline{V}_{01}} \quad (2)$$

Z1 = Z2 = Z0
e.g.Z0 = 50 Ohm



$$\underline{S}_{12} = 2 * \frac{\underline{V}_1}{\underline{V}_{02}} \quad (3)$$

$$\underline{S}_{22} = 2 * \frac{\underline{V}_2}{\underline{V}_{02}} - 1 \quad (4)$$

SMALL SIGNAL AND LARGE SIGNAL S-PARAMETERS

So far, we introduced the S-parameters and compared them to the other twoport parameters like Y or Z. This means, S-parameters are *small signal* parameters by definition. For a transistor as an example, the S-parameters do *not* reflect non-linear amplification phenomena like compression etc.

In general, twoport parameters of non-linear components like transistors or diodes vary as a function of input power. Referring to S-parameters, the parameters $|S_{11}|^2$ and $|S_{21}|^2$ are defined as a function of power incident at port 1 with no power incident at port 2; whereas $|S_{12}|^2$ and $|S_{22}|^2$ are defined as a function of power incident at port 2. Therefore, in RF simulators, all S-parameters of nonlinear electrical elements are represented by a 2x2 S-parameter matrix versus input power at port 1 for forward and port 2 for reverse operation.

This data set has been accepted as a convenient means of characterizing nonlinear devices by their large-signal S-parameters and have been successfully used for designing power amplifiers, oscillators, etc.

However, keep in mind that when the S-parameters become RF-power dependent, harmonics occur. In order to characterize these harmonic frequencies, a spectrum analyzer or a non-linear network analyzer should be applied!

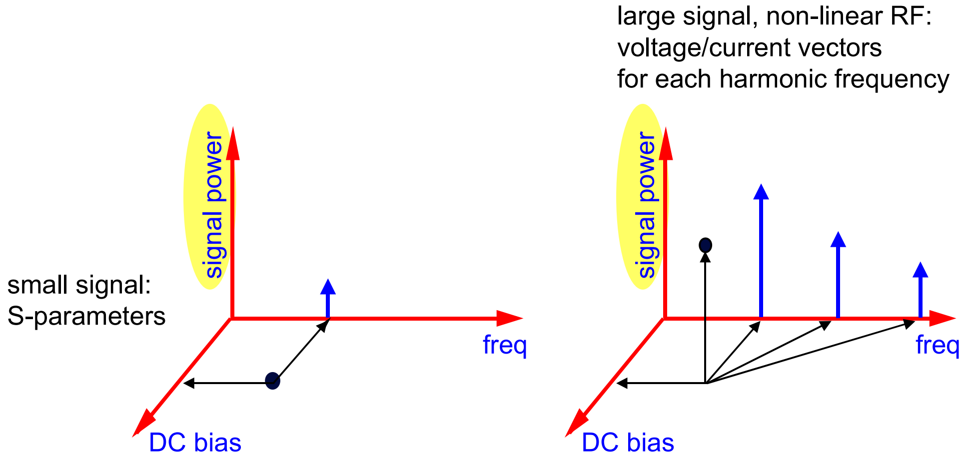


Fig.1: small signal S-parameters are a function of frequency and, for transistors, also of bias, while large signal S-parameters also cover the signal power.

Note: even for the three-dimensional character of large signal S-parameters, they are by definition linear. This means, a single frequency power $|a_1|^2$ injected into port1 will lead to reflected and transmitted power with *exactly* the same frequency. No harmonic frequencies!

Transforming 3-port parameters to 2-port and back

In order to transform e.g. Common-Emitter S-Parameters to Common-Base, we will now consider 3-Port S-Parameters. Provided a certain port is connected to ground, we are then able to evaluate the conversion formulas.

Note: for details, see the publication /Stassen/ mentioned at the end of this chapter.

3-PORT -> 2-PORT

Provided that the device under test is connected with 50 Ω to all its 3 ports, the 3-pole network S-parameters are defined as below:

$$\begin{aligned} b_1 &= S_{11} * a_1 + S_{12} * a_2 + S_{13} * a_3 \\ b_2 &= S_{21} * a_1 + S_{22} * a_2 + S_{23} * a_3 \\ b_3 &= S_{31} * a_1 + S_{32} * a_2 + S_{33} * a_3 \end{aligned} \qquad (1)$$

If port 3 is connected to ground, we get for the corresponding reflection coefficient Γ_3

$$\Gamma_3 = \frac{a_3}{b_3} = -1 \qquad (2)$$

Thus we get the 2-port network parameters from the 3-port ones as:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} - \dfrac{S_{13} * S_{31}}{1+S_{33}} & S_{12} - \dfrac{S_{13} * S_{32}}{1+S_{33}} \\ S_{21} - \dfrac{S_{31} * S_{23}}{1+S_{33}} & S_{22} - \dfrac{S_{23} * S_{32}}{1+S_{33}} \end{pmatrix} * \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad (3)$$

Transforming from 2-PORT -> 3-PORT

Using the matrix in (3), we can also calculate the 3-port S-parameters out of 2-port parameters.

As described in publication /Stassen/, see below, the sum of each column and row in a 3-port S-parameter matrix equals '1', i.e.

$$\sum_{j=1}^{3} S_{ij} = 1 \quad \text{for} \quad i=1,2,3$$

and

$$\sum_{i=1}^{3} S_{ij} = 1 \quad \text{for} \quad j=1,2,3$$

Referring to the above equations (1) and (29, we can calculate the 3-port S-parameters out of the 2-port ones (with Index T (Twoport Measurement):

$$S_{33} = \frac{\sum\limits_{\substack{i=1,2\\j=1,2}} S_{ijT}}{4 - \sum\limits_{\substack{i=1,2\\j=1,2}} S_{ijT}}$$

$$S_{32} = \frac{1+S_{33}}{2}\left(1 - S_{12T} - S_{22T}\right)$$

$$S_{23} = \frac{1+S_{33}}{2}\left(1 - S_{21T} - S_{22T}\right)$$

$$S_{22} = S_{22T} + \frac{S_{23} + S_{32}}{1+S_{33}}$$

$$S_{31} = 1 - S_{33} - S_{32}$$
$$S_{13} = 1 - S_{23} - S_{33}$$
$$S_{12} = 1 - S_{22} - S_{32}$$
$$S_{11} = 1 - S_{21} - S_{31}$$
$$S_{21} = 1 - S_{22} - S_{23}$$

Once these 3-port S-parameters are known, they can be used to calculate the common-base or common-collector 2-port S-parameters of a bipolar transistor etc.

Publications:

Rudolf Stassen, 'Einsatz eines Mikrowellen-SPitzenmeßplatzes zur Charakterisierung von Transistore direkt auf dem Wafer im Frequenzbereich von 0,045 bis 26,5GHz', Diplomarbeit am Institut für Schicht- und Ionentechnik am Forschungszentrum Jülich GmbH, available from Technische Informationsbibliothek, Hannover.

Transforming from 2-PORT -> 1-PORT:
and its application for the Q factor of RF-passive components

This is required if a component has been measured in 2-Port mode, and some of its components characteristics might be affected by the characteristic impedance of the opposite VNA port. An example is the calculation of the quality factor of a spiral inductor, which is represented by

$$Q = \frac{\mathrm{IMAG}(\underline{Z}_{inductor})}{\mathrm{REAL}(\underline{Z}_{inductor})}$$

and which cannot be calculated correctly out of the 2-Port S-Parameters because of the Z0=50 Ω of Port 2.

In this case, we refer to equation. (3) from above and obtain:

$$S_{11\_1port} = S_{11} - \frac{S_{12} * S_{21}}{1 + S_{22}}$$

From that, we apply the basic S_{11} <> R conversion, mentioned in the above chapter on Smith charts,

$$\underline{R} = Z0 \cdot \frac{1 + \underline{S_{11}}}{1 - \underline{S_{11}}}$$

and obtain for the input impedance at Port 1

$$Z_{11\_1port} = Z_0 * \frac{1 + S_{11\_1port}}{1 - S_{11\_1port}}$$

From this, we can obtain the requested 1-Port characteristic like the Q factor of a spiral inductor

$$Q = \frac{\mathrm{IMAG}(\underline{Z}_{11\_1port})}{\mathrm{REAL}(\underline{Z}_{11\_1port})}$$

Note: the same result is obtained when converting the S-parameters to Y-parameters, and then calculating

$$Q = \frac{-\mathrm{IMAG}(\underline{Y}_{11})}{\mathrm{REAL}(\underline{Y}_{11})}$$

Converting S-parameters to differential/common mode parameters

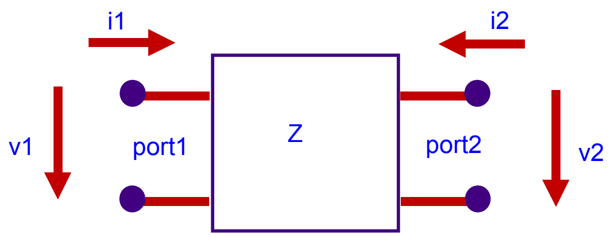
When measuring S-parameters using a VNA, the DUT is characterized by reflection and thru measurements at both ports, referring to signal ground.

In some applications, however, the device may be used in differential mode, e.g. in differential amplifier stages etc. In such a case, the characteristics of the device may be different from those measured in the conventional S-parameter measurement environment. This is especially important for passive RF components and their Q factor, i.e. the 'quality factor' of their RF performance. Therefore, it is desirable to also model both, the performance and the Q factor, in the appropriate stimulus condition.

Since the S-parameter are linear twoport parameters, they can be converted into Z-parameters, and the Z-parameters can be converted in special Z-parameters for common mode excitation and differential mode excitation.

Let's begin with the conventional Z matrix definition:

Z Matrix



For differential mode condition, we can define:

$$v\_diff = v_d = v1 - v2 \quad \text{and} \quad i\_diff = i_d = \frac{i1 - i2}{2}$$

And for common mode,

$$v\_common = v_c = \frac{v1 + v2}{2} \quad \text{and} \quad i\_common = i_c = i1 + i2$$

Referring to the Z matrix

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix},$$

we can substitute by the common and differential voltages and currents and obtain

$$\begin{bmatrix} v_c \\ v_d \end{bmatrix} = \begin{bmatrix} Z_{cc} & Z_{cd} \\ Zdc & Z_{dd} \end{bmatrix} \cdot \begin{bmatrix} i_c \\ i_d \end{bmatrix}$$

with

$$Z_{cc} = \frac{Z_{11} + Z_{12} + Z_{21} + Z_{22}}{4} \qquad Z_{cd} = \frac{Z_{11} - Z_{12} + Z_{21} - Z_{22}}{2}$$

$$Z_{dc} = \frac{Z_{11} + Z_{12} - Z_{21} - Z_{22}}{2} \qquad Z_{dd} = Z_{11} - Z_{12} - Z_{21} + Z_{22}$$

This conversion scheme can be applied to get differential mode and common mode S-parameters:

- convert S-parameters to Z
- apply common/differential conversion
- convert Z-parameters back to S

Related to the Q-Factor:
Usually, the Q factor is calculated from the Z-parameters, converted to 1-port Z-parameters as

$$Q = \frac{\mathrm{IMAG}(\underline{Z}_{11\_1port})}{\mathrm{REAL}(\underline{Z}_{11\_1port})}$$, see above.

This refers to a measurement condition where the 1st port is tested, and the 2nd is grounded.

Under differential mode operating conditions, this is not the case. Referring to the sections above, we now have:

$$L_{dd} = \frac{\mathrm{IMAG}(Z_{dd})}{2 \cdot \mathrm{PI} \cdot \mathrm{freq}} \quad \text{and} \quad R_{dd} = \mathrm{REAL}(Z_{dd})$$

what gives

$$Q_{dd} = \frac{\mathrm{IMAG}(Z_{dd})}{\mathrm{REAL}(Z_{dd})} = \frac{L_{dd} \cdot 2 \cdot \mathrm{PI} \cdot \mathrm{freq}}{R_{dd}}$$

Acknowledgments:
I would like to thank Laurent Gambus of Philips in Caën for bringing this topic of differential/common 2-port parameters to my attention and also for the basic matrix conversion method.

Publications on differential/common mode Z-parameters:

M.Danesh, J.R.Long, R.A.Hadaway, D.L.Harame, A Q-Factor Enhancement Technique For MMIC Inductors, 1998 IEEE MTT-S Digest, p.183-186